Surds

1.

Simplify

a $\sqrt{18} + \sqrt{50}$ b $\sqrt{48} - \sqrt{27}$ c $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$ e $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$ f $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

2.

Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ b $4 - \sqrt{3} - 2(1 - \sqrt{3})$ c $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$ e $(3\sqrt{3} - 4)^2$ f $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

3.

Simplify

a $\sqrt{8} + \dfrac{6}{\sqrt{2}}$ b $\sqrt{48} - \dfrac{10}{\sqrt{3}}$ c $\dfrac{6-\sqrt{8}}{\sqrt{2}}$

d $\dfrac{\sqrt{45}-5}{\sqrt{20}}$ e $\dfrac{1}{\sqrt{18}} + \dfrac{1}{\sqrt{32}}$ f $\dfrac{2}{\sqrt{3}} - \dfrac{\sqrt{6}}{\sqrt{72}}$

4.

Solve each equation, giving your answers as simply as possible in terms of surds.

a $x(x + 4) = 4(x + 8)$ b $x - \sqrt{48} = 2\sqrt{3} - 2x$

c $x\sqrt{18} - 4 = \sqrt{8}$ d $x\sqrt{5} + 2 = \sqrt{20}(x - 1)$

5.

Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{2}+1}$ b $\dfrac{4}{\sqrt{3}-1}$ c $\dfrac{1}{\sqrt{6}-2}$ d $\dfrac{3}{2+\sqrt{3}}$

e $\dfrac{1}{2+\sqrt{5}}$ f $\dfrac{\sqrt{2}}{\sqrt{2}-1}$ g $\dfrac{6}{\sqrt{7}+3}$ h $\dfrac{1}{3+2\sqrt{2}}$

i $\dfrac{1}{4-2\sqrt{3}}$ j $\dfrac{3}{3\sqrt{2}+4}$ k $\dfrac{2\sqrt{3}}{7-4\sqrt{3}}$ l $\dfrac{6}{\sqrt{5}-\sqrt{3}}$

Surds Problem Solving (Ex 1.3B Q 3, 7, 9, 14, 15)

1.

a A car travels $18\sqrt{35}$ m in $6\sqrt{7}$ s. Work out its speed, showing your working.

b A runner travels for 5 s at $\frac{8}{\sqrt{5}}$ m s^{-1}.
Work out how far she ran in simplified form. Show your working.

2.

Base camp is $5\sqrt{5}$ miles due east and $5\sqrt{7}$ miles due north of a walker. What is the exact distance from the walker to the camp? Show your working.

3.

The equation of a parabola is $y^2 = 4ax$

Find y when $a = 6 - \sqrt{6}$ and $x = \frac{6+\sqrt{6}}{10}$

Show your working and give your answer in simplified form.

4.

An equilateral triangle with side length $5\sqrt{6}$ inches has one vertex at the origin and one side along the positive x-axis.

The centre is on the vertical line of symmetry, $\frac{1}{3}$ of the way from the x-axis to the vertex.
Work out the distance from the origin to the centre of mass of the triangle. Show your working.

5.

The Indian Mathematician Brahmagupta (598 – 670) developed a formula to calculate the area of a cyclic quadrilateral.

If the sides of the quadrilateral are a, b, c and d, and $s = \frac{a+b+c+d}{2}$, the area is $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

A quadrilateral with side lengths $5+5\sqrt{2}$, $3+3\sqrt{2}$, $6+4\sqrt{2}$ and $2+4\sqrt{2}$ cm is inscribed in a circle.

Prove that $A = 2(1+\sqrt{2})\sqrt{15(11+8\sqrt{2})}$

Surds Solutions

1.

a $= 3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$ **b** $= 4\sqrt{3}-3\sqrt{3}=\sqrt{3}$ **c** $= 4\sqrt{2}+6\sqrt{2}=10\sqrt{2}$

d $= 6\sqrt{10}-4\sqrt{10}=2\sqrt{10}$ **e** $= 2\sqrt{5}-3\sqrt{5}+6\sqrt{5}=5\sqrt{5}$ **f** $= 2\sqrt{6}+5\sqrt{6}-8\sqrt{6}=-\sqrt{6}$

2.

a $=3+2\sqrt{3}$

b $=4-\sqrt{3}-2+2\sqrt{3}$
$=2+\sqrt{3}$

c $=2+\sqrt{3}+2\sqrt{3}+3$
$=5+3\sqrt{3}$

d $=4+8\sqrt{3}+\sqrt{3}+6$
$=10+9\sqrt{3}$

e $=27-24\sqrt{3}+16$
$=43-24\sqrt{3}$

f $=6\sqrt{3}-45+2-5\sqrt{3}$
$=-43+\sqrt{3}$

3.

a $=2\sqrt{2}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=2\sqrt{2}+3\sqrt{2}$
$=5\sqrt{2}$

b $=4\sqrt{3}-\frac{10}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$=4\sqrt{3}-\frac{10}{3}\sqrt{3}$
$=\frac{2}{3}\sqrt{3}$

c $=\frac{6-2\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=\frac{6\sqrt{2}-4}{2}$
$=3\sqrt{2}-2$

d $=\frac{3\sqrt{5}-5}{2\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$
$=\frac{15-5\sqrt{5}}{10}$
$=\frac{1}{2}(3-\sqrt{5})$

e $=\frac{1}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{1}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
$=\frac{1}{6}\sqrt{2}+\frac{1}{8}\sqrt{2}$
$=\frac{7}{24}\sqrt{2}$

f $=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}}$
$=\frac{2}{3}\sqrt{3}-\frac{1}{6}\sqrt{3}$
$=\frac{1}{2}\sqrt{3}$

4.

a $x^2+4x=4x+32$
$x^2=32$
$x=\pm\sqrt{32}$
$x=\pm4\sqrt{2}$

b $x-4\sqrt{3}=2\sqrt{3}-2x$
$3x=6\sqrt{3}$
$x=2\sqrt{3}$

c $3\sqrt{2}\,x-4=2\sqrt{2}$
$6x-4\sqrt{2}=4$
$6x=4+4\sqrt{2}$
$x=\frac{2}{3}(1+\sqrt{2})$

d $\sqrt{5}\,x+2=2\sqrt{5}\,(x-1)$
$5x+2\sqrt{5}=10(x-1)$
$5x=10+2\sqrt{5}$
$x=2+\frac{2}{5}\sqrt{5}$

5.

a $=\dfrac{1}{\sqrt{2}+1}\times\dfrac{\sqrt{2}-1}{\sqrt{2}-1}=\dfrac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

b $=\dfrac{4}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$

c $=\dfrac{1}{\sqrt{6}-2}\times\dfrac{\sqrt{6}+2}{\sqrt{6}+2}=\dfrac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$

d $=\dfrac{3}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}=\dfrac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$

e $=\dfrac{1}{2+\sqrt{5}}\times\dfrac{2-\sqrt{5}}{2-\sqrt{5}}=\dfrac{2-\sqrt{5}}{4-5} = \sqrt{5}-2$

f $=\dfrac{\sqrt{2}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}=\dfrac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2+\sqrt{2}$

g $=\dfrac{6}{\sqrt{7}+3}\times\dfrac{\sqrt{7}-3}{\sqrt{7}-3}=\dfrac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$

h $=\dfrac{1}{3+2\sqrt{2}}\times\dfrac{3-2\sqrt{2}}{3-2\sqrt{2}}=\dfrac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

i $=\dfrac{1}{4-2\sqrt{3}}\times\dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}=\dfrac{4+2\sqrt{3}}{16-12} = \frac{1}{2}(2+\sqrt{3})$ or $1+\frac{1}{2}\sqrt{3}$

j $=\dfrac{3}{3\sqrt{2}+4}\times\dfrac{3\sqrt{2}-4}{3\sqrt{2}-4}=\dfrac{3(3\sqrt{2}-4)}{18-16} = \frac{3}{2}(3\sqrt{2}-4)$ or $\frac{9}{2}\sqrt{2}-6$

k $=\dfrac{2\sqrt{3}}{7-4\sqrt{3}}\times\dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}=\dfrac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$

l $=\dfrac{6}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\dfrac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$

Surds Problem Solving Solutions

1.

a $3\sqrt{5}\ \text{m s}^{-1}$ **b** $8\sqrt{5}\ \text{m}$

2.

$10\sqrt{3}\ \text{m}$

3.

$2\sqrt{3}$

4.

13

Thus coordinates of the centroid are $\left(\frac{5\sqrt{6}}{2}, \frac{5\sqrt{2}}{2}\right)$, so the distance from the origin to the centroid $= 5\sqrt{2}$

5.

$$2s = 5 + 5\sqrt{2} + 3 + 3\sqrt{2} + 6 + 4\sqrt{2} + 2 + 4\sqrt{2} = 16 + 16\sqrt{2}$$

$$s = 8 + 8\sqrt{2}$$

$$A^2 = (3 + 3\sqrt{2})(5 + 5\sqrt{2})(2 + 4\sqrt{2})(6 + 4\sqrt{2})$$

$$= 3(1 + \sqrt{2})5(1 + \sqrt{2})2(1 + 2\sqrt{2})2(3 + 2\sqrt{2})$$

$$= 2^2(1 + \sqrt{2})^2 15(11 + 8\sqrt{2})$$

$$A = 2(1 + \sqrt{2})\sqrt{15(11 + 8\sqrt{2})}$$

(Full worked solutions on Kerboodle Ex1.3B Q 3,7,9,14,15)